Instability, Complexity, and Throughputs In Landscape Evolution

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Theory

Consider a landscape or geomorphic system in terms of its mass and energy throughputs *T*. The landscape consists of i = 1, 2, . . ., n components, each with their own mass and energy inputs and outputs such that $T = \sum T_i$. Throughput is controlled by inputs (g) and outputs (f) to each component, and changes in storage (Δs),

$$T_i = g_i - f_i + \Delta s_i \tag{1}$$

The proportion of throughput associated with each component (Q) is

$$Q_i = T_i / T \tag{2}$$

The maximum uncertainty or complexity of the fluxes in the system can be measured using the Shannon entropy:

$$H = -\sum Q_i \ln Q_i \tag{3}$$

The mass and energy fluxes can be divided into external inputs (flows into one or more *i*), external exports (flows from one or more *i* to the external environment), and internal flows between components.

In information theory terms the decrease in uncertainty from knowing the external inputs is given by

$$I_o = T \sum_i g_{ei} Q_i \ln[g_{ei}/(\sum g_{ei} Q_j)] \tag{4}$$

where g_{ei} is the proportion of the input to *i* coming from outside the system.

A similar consideration of the internal flux exchanges is sometimes termed integrality, or when applied to ecosystem studies, mutual independence:

$$I = T \sum_i \sum_k g_{ki} Q_i \ln[g_{ki}/(\sum g_{kj} Q_j)] \tag{5}$$

where g_{ki} is the probability that flux at *i* comes directly from *k*.

The analog of eq. (4) for exports of usable mass and energy (i.e., excluding energy dissipated as heat) is

$$A_o = T \sum_J f_{je} Q_i \ln[f_{je}/(\sum f_{ei} Q_j)] \tag{6}$$

The proportion of outflow from component *j* to the external environment is f_{je}.

If the probability of any quantity of flow leaving component *i* directly contributing to component *j* is f_{ij}, then a measure of mutual sustenance is

$$A = T \sum_k \sum_j f_{ij} Q_i \ln[f_{ij} / (\sum_i f_{ij} Q_j)] \tag{7}$$

Note that equations (5) and (7) differ by their attention to the probability of inputs coming from a given component (5), versus the likelihood of outputs being directed to a given component (7).

In the ecological literature *A* is referred to as ascendancy, relating to (for example) the complexity and interdependency of ecosystems (e.g. Ulanowicz, 1980).

The relationship between *A* and other parameters is

$$A = H - (S + R + A_o) \tag{8}$$

Where S is a measured of unfilled mass/energy flux potential:

$$S = (I + I_o) - (A + A_o) \tag{9}$$

R is a measure of redundancy,

$$R = H - (I + I_o) \tag{10}$$

These inequalities hold:

$$H \geq (I + I_o) \geq (A + A_o) \geq 0 \qquad (11)$$

From eq. (8) we can see that

$$\Delta A/\Delta t = \Delta H/\Delta t - \Delta S/\Delta t - \Delta R/\Delta t - \Delta A_o/\Delta t \qquad (12)$$

In a dynamical system, the change in Shannon entropy over time is equal to the Kolmogorov entropy,

$$K = \Delta H/\Delta t \qquad (13)$$

K-entropy is also the sum of the positive Lyapunov exponents (λ) of a dynamical system, where an *n*-component system has *n* exponents such that $\lambda_1 > \lambda_2 > . . . > \lambda_n$. Because dynamical instability and chaos is indicated by the presence on any positive Lyapunov exponent ($\lambda_1 > 0$), positive K-entropy that increases in ascendancy may be associated with dynamical instability. Chaos and instability ($\Delta H/\Delta t > 0$) is not the only way that ascendancy can increase over time, as changes in *S* (unfilled storage/flux potential), *R* (redundancy) and usable exports A_o could be negative. However, this analysis shows how nonlinear complexity and divergent evolution (i.e., dynamical instability) may play a role in the ascendant development of environmental systems.